Exploring Diagonals of Quadrilaterals

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will demonstrate knowledge and understanding of the nature of different quadrilaterals based directly on the properties of their diagonals.
- Students will identify quadrilaterals given only properties of their diagonals.

Vocabulary

- diagonal
- vertex angles
- perpendicular bisector

About the Lesson

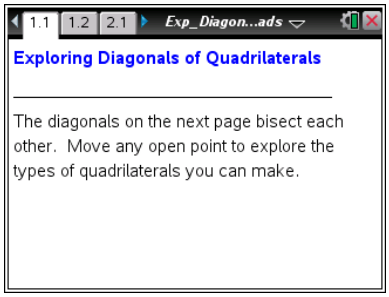
- This lesson involves manipulating endpoints and/or the intersection point of two diagonals that have special characteristics.
- As a result, students will:
  - Observe whether the two diagonals mutually bisect each other, bisect vertical angles, or are congruent, or whether one is the perpendicular bisector of the other.
  - Determine which special quadrilaterals *can* be formed and, in some cases, determine which special quadrilateral *must* be formed, based on what they know about the diagonals.

TI-Nspire™ Navigator™

- Send the TI-Nspire document to students.
- Use of Class Capture and Quick Poll will allow the teacher to assess student understanding of the TI-Nspire document and the mathematics involved in the activity.
- Use Teacher Edition computer software to review student documents.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Exp_Diagonls__of_Quads_Student.pdf
- Exp_Diagonals_of_Quads_Student.doc

TI-Nspire document

- Exp_Diagonals_of_Quads.tns

Discussion Points and Possible Answers

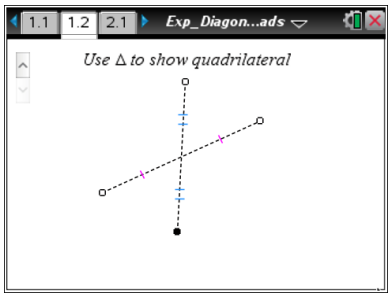
Teacher Tip: There are ongoing discussions about some definitions, particularly a *kite* and a *trapezoid*. Is a rhombus a kite? Is a trapezoid a parallelogram? Some resources say yes, and others say no. This might be a good discussion to have with students. For this activity, a rhombus is a kite, but a trapezoid is not a parallelogram. If your book is different, adjust the activity accordingly. In this activity, the following definitions are used:

- **parallelogram:** a quadrilateral with opposite sides parallel
- **rhombus:** a parallelogram with all sides congruent
- **kite:** a quadrilateral with two pairs of adjacent sides congruent
- **trapezoid:** a quadrilateral with one pair of opposite sides parallel

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☜) getting ready to grab the point. Then press ctrl to grab the point and close the hand (☜).

TI-Nspire™ Navigator™ Opportunity: *Class Capture and/or Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. The two diagonals on this page are special because they always bisect each other. Drag any open point to make a quadrilateral. To see the quadrilateral, select ▲ on the screen. Drag a point. To see angle measurements or side lengths, select ▲. Then drag a point.

Tech Tip: Students may need to click the ▲ arrow twice to see the quadrilateral for the first time.

If students are having trouble grabbing points after showing the quadrilateral or the angle or side measurements, instruct them to press esc before attempting to grab the points.

a. Try to form each of the quadrilaterals in the table below. Record your findings in the table.

Answer: Complete table is below.

	Parallelogram (not rectangle, not rhombus)	Rectangle (not square)	Rhombus (not square)	Kite (not rhombus)	Square	Trapezoid	Quadrilateral with four different side lengths
Yes or No?	**yes**	**yes**	**yes**	**no**	**yes**	**no**	**no**
Why or Why Not?	**It can be a parallelogram that is neither a rectangle nor a rhombus because I can get opposite sides parallel without having all 4 sides congruent and without having 4 right angles.**	**It can be a rectangle that is not a square because I can get 4 right angles. I do not have to make all 4 sides congruent. I just have to have opposite sides congruent.**	**It can be a rhombus that is not a square because I can get all 4 sides congruent without having right angles.**	**It can't be a kite that isn't a rhombus because I can't get 2 *distinct* pairs of adjacent sides congruent.**	**It can be a square because I can get all 4 sides congruent and 4 right angles.**	**It can't be a trapezoid because I cannot get only one pair of opposite sides parallel.**	**I always have opposite sides congruent.**

Teacher Tip: When it applies, you may want students to tell you why they know the lines are parallel using angle properties.

b. What special quadrilaterals can be formed?

Answer: Parallelogram (not rectangle, not rhombus); Rectangle (not square); Rhombus (not square); Square

Each of the remaining problems in the file contains two diagonals that have some special property. Move through pages 2.1, 2.2, 3.1, 3.2, 4.1, 4.2, 5.1, and 5.2.

2. For each problem, what special quadrilaterals can be formed? Record your findings in the appropriate rows in the table below.

Answer: Complete table is below.

Page	Diagonal Properties	Parallelogram (not rectangle, not rhombus)	Rectangle (not square)	Rhombus (not square)	Kite (not rhombus)	Square	Trapezoid	Quadrilateral with four different side lengths
1.2	*Diagonals bisect each other*	**yes**	**yes**	**yes**	**no**	**yes**	**no**	**no**
2.2	*One diagonal is a perpendicular bisector of the other*	**no**	**no**	**yes**	**yes**	**yes**	**no**	**no**
3.2	*Diagonals bisect vertex angles*	**no**	**no**	**yes**	**no**	**yes**	**no**	**no**
4.2	*Diagonals are congruent*	**no**	**yes**	**no**	**no**	**yes**	**yes***	**yes**
5.2	*Diagonals are perpendicular*	**no**	**no**	**yes**	**yes**	**yes**	**yes**	**yes**

Teacher Tip: While a trapezoid can be created with congruent diagonals, not all trapezoids have that property. Make sure students understand the difference between "can be a quadrilateral" and "must be this type of quadrilateral."

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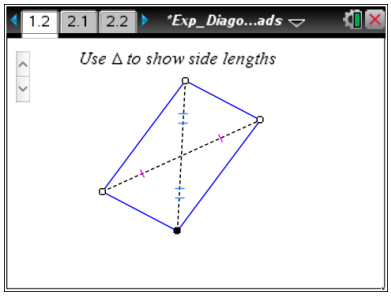

3. a. If the diagonals bisect each other, then the quadrilateral *must* be what type of figure?

Answer: Parallelogram

b. Justify your answer.

Answer: Rectangles, rhombi, and squares are all parallelograms.

Teacher Tip: Normally, the theorem in books states: If a quadrilateral is a parallelogram, then the diagonals are bisectors of each other. Now, this gives the converse: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
Depending on your students' background and the goals of your course, you could prove this statement. One method of proof would use the fact that the diagonals form two pairs of congruent triangles (SAS). Thus, since corresponding parts of congruent triangles are congruent, you would have alternate interior angles congruent. Therefore, both pairs of opposite sides would be parallel (and also congruent).

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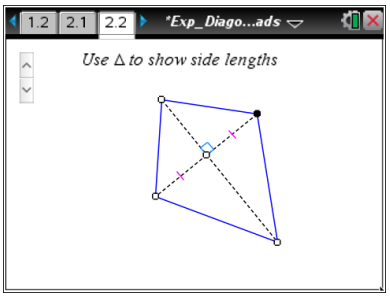

4. a. If one diagonal is the perpendicular bisector of the other, then the quadrilateral *must* be what type of figure?

Answer: Kite

b. Justify your answer.

Answer: Squares and rhombi are kites.

Teacher Tip: Normally, the theorem in books states: If a quadrilateral is a kite, then one diagonal is the perpendicular bisector of the other. Now, this gives the converse: If one diagonal is the perpendicular bisector of the other, then the quadrilateral is a kite.

Depending on your students' background and the goals of your course, you could prove this statement. One method of proof would use the fact that the diagonals form two pairs of congruent triangles (HL). Thus, since corresponding parts of congruent triangles are congruent, you would have the hypotenuses congruent. Or in this case, you would have two pairs of adjacent sides congruent.

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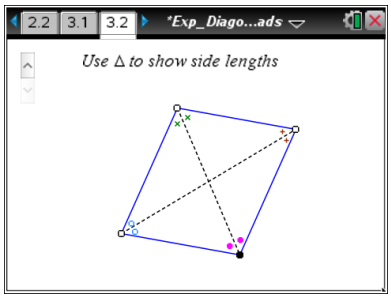

5. a. If the diagonals of a quadrilateral bisect its vertex angles, then the quadrilateral *must* be what type of figure?

Answer: Rhombus

b. Justify your answer.

Answer: A square is a rhombus.

Teacher Tip: Normally, the theorem in books states: If a quadrilateral is a rhombus, then the diagonals bisect the vertex angles. Now, this gives the converse: If the diagonals of a quadrilateral bisect the vertex angles, then the quadrilateral is a rhombus.
Depending on your students' background and the goals of your course, you could prove this statement. One method of proof would consider the triangles formed by one diagonal. In this case, these triangles would be congruent by ASA. Thus, the vertex angles contained in these two triangles would be congruent by the fact that corresponding parts of congruent triangles are congruent. Therefore, the angles formed by bisecting those two vertex angles would have to be congruent. You could arrive at a similar relationship by using the other diagonal. Then you will be able to prove all four triangles formed by the two intersecting diagonals are congruent by ASA. Therefore, all sides of the quadrilateral must be congruent and you will be able to show opposite sides are parallel as well.

Teacher Tip: Possible extensions for students could include asking students to complete similar if/then statements by adding extra stipulations on the quadrilateral. For example, for the last two rows in the table (pages 4.2 and 5.2), students might determine the following:

- If the quadrilateral is a parallelogram with congruent diagonals, then it must be a rectangle (but it can be a square).
- If the quadrilateral is a parallelogram and its diagonals are perpendicular, then it must be a rhombus (but it can be a square).

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- If the diagonals of a quadrilateral are mutually bisecting (bisect each other), then the quadrilateral *must* be a parallelogram but *can* also be a special parallelogram, including a rectangle, a rhombus, or a square.
- If in a quadrilateral one diagonal is the perpendicular bisector of the other diagonal, then the quadrilateral *must* be a kite but *can* also be a special parallelogram, such as a rhombus or a square.
- If the diagonals of a quadrilateral bisect its vertex angles, then the quadrilateral *must* be a rhombus but *can* also be a square that is a special parallelogram.

Extension: Non-Intersecting Diagonals

Although this activity deals with diagonals always having to intersect, you also may want to discuss non-intersecting diagonals with students. What kinds of quadrilaterals would have non-intersecting diagonals? Could diagonals be perpendicular and not intersect? For example, two diagonals that are perpendicular to each other but do not intersect can define a concave kite. Likewise, two equal diagonals that do not intersect define a concave general quadrilateral.

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Note 1

Entire Student Worksheet, Name of Feature: Class Capture

Use Class Capture as students complete the worksheet to determine students who are having difficulty and help them accordingly. Refresh the screen often.

Note 2

Wrap Up, Name of Feature: Quick Poll

Use Quick Poll to assess students' understanding of the Wrap Up statements.

- You might use Open Response or orally ask students to complete the following:
  - If the diagonals of a quadrilateral bisect each other, then the quadrilateral must be a _____.
- You might use True/False:
  - If in a quadrilateral one diagonal is the perpendicular bisector of the other diagonal, then the quadrilateral must be a square.
- You might use Always/Sometimes/Never:
  - If the diagonals of a quadrilateral bisect its vertex angles, then the quadrilateral will be a square.